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# A Convenient, Explicit Formula for Oblique-Shock Calculations

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THIS Note presents an approximate formula enabling direct calculation of changes in flow properties across an oblique shock wave, which has been found to be quite useful by the author. The need for such an explicit relationship (in terms of the approach Mach number and the deflection angle) has been pointed out many times (cf. Ref. 1) and several approximate methods have been developed to fulfill this need. Among these are the Busemann power series (cf. Ref. 1), the first two terms of which are referred to as the supersonic linear and second-order theory, respectively, and several hypersonic approximations.[2-5] The formula presented in this Note is more accurate than any of these and serves as a good starting point for an iterative solution or as a fairly accurate engineering solution for systems or preliminary design work where an explicit equation is necessary.

By neglecting one term in the exact cubic equation for $\sin^2\theta$, Cleary and Axelson[3] arrived at the following relation:

$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma\sin^2\delta - \cos\delta[\tfrac{1}{4} - 1/M^2 - \gamma^2\sin^2\delta/4]^{1/2} \quad (1)$$

where $\theta$ is the shock-wave angle and $\delta$ is the deflection angle. (All other flow properties may be calculated conveniently from $\sin^2\theta$, cf. Ref. 1.) If the neglected term were retained, we would obtain the following exact (but implicit) equation:

$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma\sin^2\delta - \cos\delta[\tfrac{1}{4} - 1/M^2 - \gamma^2\sin^2\delta/4 + 1/(M^4\sin^2\theta)]^{1/2} \quad (2)$$

where only the final term differs from Eq. (1).

If we look upon Eq. (2) as a successive approximation scheme, we see that the Cleary and Axelson formula [Eq. (1)] amounts to taking $1/M^4\sin^2\theta = 0$ as a first guess. By substituting a more accurate initial guess, we can obtain a much better approximation. For this purpose, the modified hypersonic approximation of Bertram and Cook was selected. This approximation was found to be generally more accurate than other existing approximations, such as the modified hypersonic equation of Van Dyke[5] or the first- or second-order supersonic theory. Bertram and Cook's approximation can be written as follows:

$$\sin^2\theta = \left\{\frac{\gamma+1}{4}\frac{M}{\beta}\sin\delta + \left[\frac{1}{M^2} + \left(\frac{\gamma+1}{4}\frac{M}{\beta}\sin\delta\right)^2\right]^{1/2}\right\}^2 \quad (3)$$

Received March 19, 1968.

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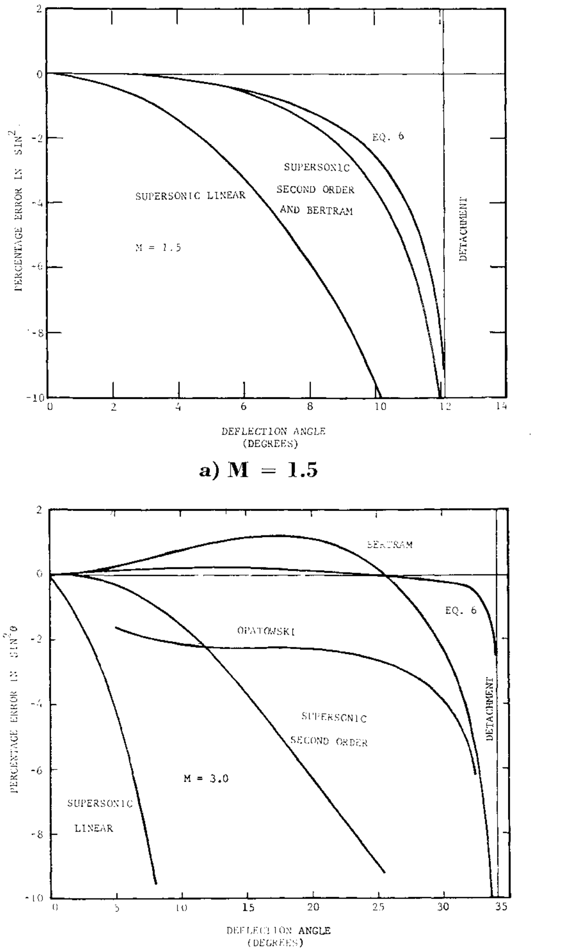



a) M = 1.5

b) M = 3.0

**Fig. 1 Accuracy of various approximations vs deflection angle.**

where $\beta = (M^2 - 1)^{1/2}$. Substituting this into the last term of Eq. (2), that term becomes

$$\frac{1}{(M^4\sin^2\theta)} = M^{-2}\left\{\frac{\gamma+1}{4}\frac{M^2}{\beta}\sin\delta + \left[1 + \left(\frac{\gamma+1}{4}\frac{M^2}{\beta}\right)^2\right]^{1/2}\right\}^{-2} \quad (4)$$

For ease in writing the final equation, and to facilitate hand calculations, we define

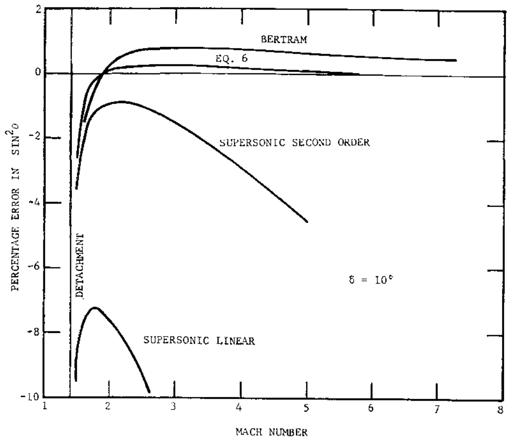
$$A \equiv [(\gamma+1)/4](M^2/\beta)\sin\delta \quad (5)$$

Equation (2) now becomes

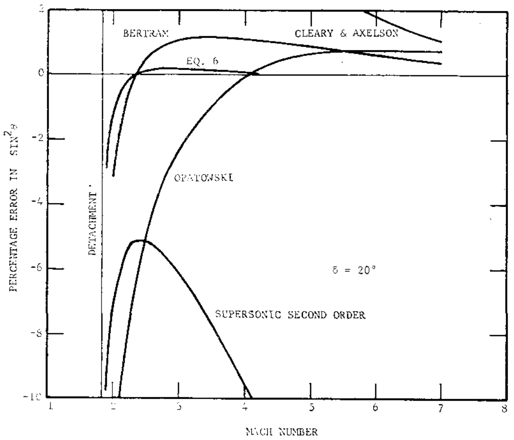
$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma\sin^2\delta - \cos\delta\{\tfrac{1}{4} - 1/M^2 - \gamma^2\sin^2\delta/4 + M^{-2}[A + (1 + A^2)^{1/2}]^{-2}\}^{1/2} \quad (6)$$

which is the desired approximate formula.

The accuracy of this approximation is compared to that of several other approximations in Figs. 1 and 2. In these figures, the supersonic linear and second-order theories for $\sin^2\theta$ were obtained by applying the exact relationship between $\sin^2\theta$ and pressure ratio to the series for pressure ratio given in Ref. 1. The percentage error in pressure ratio is almost identical to that in $\sin^2\theta$. The error in pressure coefficient varies with Mach number and deflection angle, i.e., at low pressure ratios the percentage error in $C_p$ can be several times the error in pressure ratio. The relative accuracy of the various methods is the same, however.



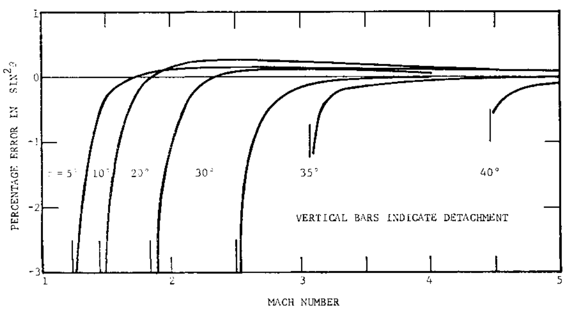

**a) Deflection angle of 10°**



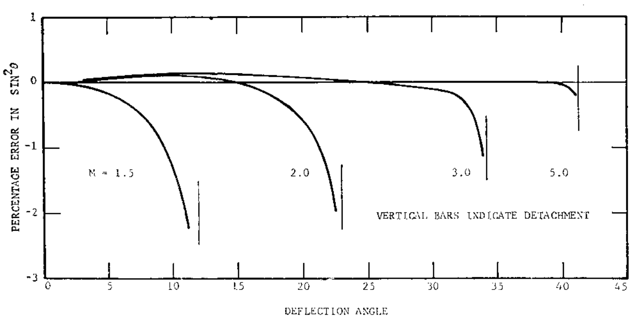

**b) Deflection angle of 20°**

**Fig. 2 Accuracy of various approximations vs Mach number.**

In Fig. 1a, at $M = 1.5$, Eq. (6) is seen to be slightly better than the supersonic second-order theory and Bertram's hypersonic theory, which are practically coincident, and much better than the supersonic linear theory. Approximations such as those of Cleary and Axelson or Opatowski are off-scale on this figure. At $M = 3$ (Fig. 1b), Eq. (6) is seen to be greatly superior to any of the other approximations. In Fig. 2, Eq. (6) is seen to be better than the other approximations for 10° and 20° deflection angles. Figure 3 indicates that Eq. (6) is quite accurate, except very close to detachment, even for very large deflection angles. Further improvement at the lower Mach numbers is possible through the use of an additional iteration formula applied to Bertram's equation before substitution into Eq. (2). However, the added complexity of the resulting equations is not justified by the improvement gained.




**a) vs Mach number**




**b) vs deflection angle**

**Fig. 3 Accuracy of present approximation.**

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# VS/TOL All-Weather Guidance

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In spite of some problems still unresolved, there is every reason to believe that an economically viable V/STOL aircraft will be developed and will serve as a competitive medium of intercity short-haul transport in the 1975–1980 time period. The success of V/STOL vehicles to fill a mass transportation role depends heavily on their ability to operate under all weather conditions.

Analyses and studies such as those made by McDonnell Aircraft Company[1] and Stanford Research Institute[2] provide useful guidelines on the nature of typical intercity operations and furnish a tentative set of requirements for an all-weather guidance system. They show that the greatest commercial potential for V/STOL aircraft is on intercity routes of less than 500 miles. Requirements for V/STOL airport design, air traffic control and navigation system, and approach and landing aids are linked with such factors as the economics of urban land values, flow of other traffic modes, location of ports, vehicle characteristics, and the mix of other air traffic.

### Requirements

For maximum utility, V/STOL aircraft must operate close to downtown areas, using ports located alongside the waterfront or other open areas surrounded by obstructions. To be economically competitive, a V/STOL transportation system must also achieve schedule reliability equivalent to or better than conventional aircraft. Providing service under instrument conditions, making optimum use of low-altitude airspace, and taking full advantage of the flight characteristics

Presented as Paper 67-796 at the AIAA 4th Annual Meeting and Technical Display, Anaheim Calif., October 23–27, 1967; submitted October 6, 1967; revision received April 12, 1968.

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